Binomische Formeln :

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b) \cdot (a+b)$$

$$x^2 - 1$$

$$x^2 - 1^2 = (x-1) \cdot (x+1)$$

Potenzregeln :

$$a^2 + a^2 + a = 2a^2 + a$$

$$3b^2 - b - 1b^2 - 2 = 2b^2 - b - 2$$

Text

$$a^m \cdot a^n = a^{m+n} \Rightarrow a^5 \cdot a^4 = a^9$$

$$a^n \cdot b^n = (a \cdot b)^n \Rightarrow 5^4 \cdot 6^4 = (5 \cdot 6)^4$$

$$\frac{a^n}{a^m} = a^{n-m} \Rightarrow \frac{5^3}{5^4} = 5^{-1} = \frac{1}{5}$$

$$5^{-2} = \frac{1}{5^2}$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$(a^n)^m = a^{n \cdot m} \Rightarrow (5^4)^2 = 5^8$$

$$\left(\sqrt[n]{a}\right)^n = a \quad \text{mit } a \geq 0$$

$$\sqrt[5]{a} = a^{\frac{1}{5}} \qquad \Big| \qquad \sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[n]{a} \cdot \sqrt[m]{a} = \sqrt[n \cdot m]{a^{n+m}}$$

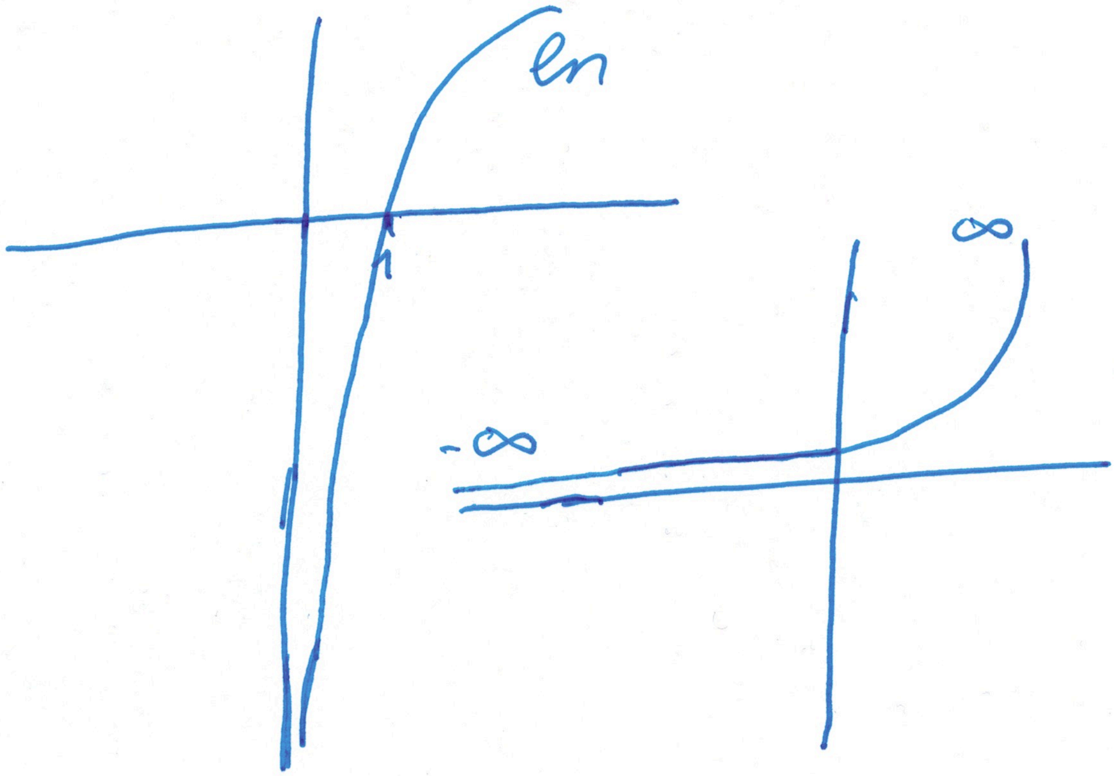
$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$$

$$\sqrt[3]{\sqrt[2]{5}} = \sqrt[3 \cdot 2]{5} = \sqrt[6]{5}$$

ln regeln:



$e^{\frac{1}{x}}$ $x \to \infty$

$e^0 = 1$

$e^\infty = \infty$

$e^{-\infty} = 0$

$\ln(1) = 0$

$\ln(x)$ $1 > x > 0$

negativ $(-\infty)$

$x > 1$

$\cancel{\ln}(\cancel{e}) = 1$

$\cancel{e}^{\cancel{\ln}(x)} = x$

$$\ln(5\cdot x) = \ln(5) + \ln(x)$$

$$\ln(5) + \ln(x) = \ln(5x)$$

$$\ln\left(\frac{5}{x}\right) = \ln(5) - \ln(x)$$

$$\ln(5) - \ln(x) = \ln\left(\frac{5}{x}\right)$$

$$\ln(5x^2) = \ln(5) + \ln(x^2)$$
$$= \ln(5) + 2\ln(x)$$

$$\ln(x^2) = 2\cdot\ln(x)$$

$$\ln(5x+2) = \ln\left(x\cdot\left(5+\frac{2}{x}\right)\right)$$
$$= \ln(x) + \ln\left(5+\frac{2}{x}\right)$$

$$\ln(5x^2-3) = \ln\left(x^2\left(5-\frac{3}{x^2}\right)\right)$$
$$= \ln(x^2) + \ln\left(5-\frac{3}{x^2}\right)$$
$$= 2\ln(x) + \ln\left(5-\frac{3}{x^2}\right)$$

$$2\ln(x) + \ln\left(5 - \frac{3}{x^2}\right) \quad | \ln(x)$$

$$2\frac{\ln(x)}{\ln(x)} + \frac{\ln\left(5 - \frac{3}{x^2}\right)}{\ln(x)}$$

$$2\ln(x) + \ln\left(5 - \frac{3}{x^2}\right) \quad | x^2$$

$$\frac{2}{x^2} \cdot \ln(x) + \frac{\ln\left(5 - \frac{3}{x^2}\right)}{x^2}$$

$$\underbrace{\frac{2}{x} \cdot \frac{\ln(x)}{x}}_{x^2} + \frac{\ln\left(5 - \frac{3}{x^2}\right)}{x^2}$$

Quadratische ergänzung:

Vollständiges Video:

https://www.youtube.com/watch?v=0KoPFqqVkkY

$$x^2 - 6x + 1$$

$$x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 1$$

$$\left(x - \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 1$$

$$\left(x - \frac{3}{1}\right)^2 - 9 + 1$$

$$(x-3)^2 - 8$$

$$(x-3)^2 - 8 = 0$$

$$(x-3)^2 = 8$$

$$|x-3| = \sqrt{8}$$

$$x - 3 = \sqrt{8}$$

$$x = \sqrt{8} + 3$$

$$-|x-3| = \sqrt{8}$$

$$-x + 3 = \sqrt{8} \quad | \cdot (-1)$$

$$-x = \sqrt{8} - 3$$

$$x = -\sqrt{8} + 3$$

2 Beispiel:

$$x^2 - 1x + 1$$

$$x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$$

ohne ²

$$\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$$

$$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{1}{1}_{\cdot 4}$$

$$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{4}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \neq 0$$

$$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$$

$$\left|x - \frac{1}{2}\right| = \sqrt{-\frac{3}{4}}$$

PQ Formel:

$$+\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x^2 \underbrace{-6}_{p} x \underbrace{+1}_{q}$$

gegenteil gegenteil

$$+\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - 1}$$

$$+\frac{6}{2} + \sqrt{\left(\frac{6}{2}\right)^2 - 1} = 3 + \sqrt{9-1} = 3 + \sqrt{8}$$

$$+\frac{6}{2} - \sqrt{\left(\frac{6}{2}\right)^2 - 1} = 3 - \sqrt{9-1} = 3 - \sqrt{8}$$

Beträge auflösen:

$$\frac{1}{\ln|x-e|} \neq 0$$

$\ln(1)=0$
$\ln(0)=$ nicht definiert

Vollständiges Video:

https://www.youtube.com/watch?v=76s0FPGKMpo

$$\frac{1}{\ln|x-e|}$$

$$\ln\underbrace{|x-e|}_{\substack{1\\0}} \neq 0$$

$|x-e| \neq 1$

1 Fall: $x-e \neq 1$

$\boxed{x = 1+e}$

2 Fall:

$-|x-e| \neq 1$

$-x+e \neq 1$

$-x \neq 1-e \quad |\cdot(-1)$

$\boxed{x \neq -1+e}$

$|x-e| \neq 0$

1 Fall:

$x-e \neq 0$

$\boxed{x \neq e}$

2 Fall:

$-|x-e| \neq 0$

$-x+e \neq 0$

$-x \neq -e \quad |\cdot(-1)$

$\boxed{x \neq e}$

$$|+x^2-1|=0$$

1 Fall

$$x^2-1=0$$

$$x^2=1$$

$$|x|=\sqrt{1}$$

1 Fall 1a:

$$x=\sqrt{1}$$

$$\boxed{x=1}$$

2 Fall 2a:

$$-x=\sqrt{1}$$

$$-x=1 \quad |\cdot(-1)$$

$$x=-1$$

$$-x \geq 1 \quad |\cdot(-1)$$

$$\boxed{x \leq -1}$$

beisp_ - - - - . . .

2 Fall:

$$-x^2+1=0$$

$$-x^2=-1 \quad |\cdot(-1)$$

$$x^2=+1$$

~~1 Fall 1a~~

$$|x|=\sqrt{1}$$

1 Fall 1a

$$x=\sqrt{1}$$

$$\boxed{x=1}$$

2 Fall 1b:

$$-x=\sqrt{1}$$

$$-x=1 \quad |\cdot(-1)$$

$$\boxed{x=-1}$$

$$\left(x-\frac{1}{2}\right)^2+\frac{3}{4}>1$$

$$\left(x-\frac{1}{2}\right)^2>\frac{1}{1}-\frac{3}{4}$$

$$\left(x-\frac{1}{2}\right)^2>\frac{4}{4}-\frac{3}{4}$$

$$\left(x-\frac{1}{2}\right)^2>\frac{1}{4}$$

$$\left|x-\frac{1}{2}\right|>\sqrt{\frac{1}{4}}$$

1 Fall:

$$x-\frac{1}{2}>\sqrt{\frac{1}{4}}$$

$$x>\sqrt{\frac{1}{4}}+\frac{1}{2}$$

$$x>\frac{1}{2}+\frac{1}{2}$$

$$\boxed{x>1}$$

2 Fall:

$$-x+\frac{1}{2}>\sqrt{\frac{1}{4}}$$

$$-x>\sqrt{\frac{1}{4}}-\frac{1}{2}$$

$$-x>\frac{1}{2}-\frac{1}{2}$$

$$-x>0 \quad |\cdot(-1)$$

$$x<-0$$

$$\boxed{x<0}$$

Linearfaktorzerlegung

$$\left(\frac{(n^2+3n+2)}{(n^2+4n-5)}\right)^n \qquad +\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Vollständiges Video:

https://www.youtube.com/watch?v=SzDmILSle1w

$$n^2 \underbrace{+3}_{p} n \underbrace{+2}_{q}$$

$$-\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 2}$$

$$-\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{2}{1}}$$

$$-\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{8}{4}}$$

$$\rightarrow -\frac{3}{2} + \sqrt{\frac{1}{4}} = -\frac{3}{2} + \frac{1}{2} = \frac{-2}{2} \boxed{= -1}$$

$$-\frac{3}{2} - \sqrt{\frac{1}{4}} = -\frac{3}{2} - \frac{1}{2} = -\frac{4}{2} = \frac{2}{1} \cancel{\boxed{= -1}}$$

$$= \boxed{-2}$$

$$n^2 + 4n - 5$$

$$-\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 5}$$

$$-2 \pm \sqrt{4+5}$$

$$-2 + \sqrt{9} = -2 + 3 = \boxed{1}$$

$$-2 - \sqrt{9} = -2 - 3 = \boxed{-5}$$

$$\Rightarrow \left(\frac{(n+1)\cdot(n+2)}{(n-1)\ (n+5)}\right)^n$$

Mit dem dritten Binom erweitern:

$$a^2 - b^2 = (a+b)\cdot(a-b)$$

$$\underbrace{\sqrt{a-b}}_{a} - \underbrace{\sqrt{x+y}}_{b}$$

$$= \frac{(\underbrace{\sqrt{a-b}}_{a} - \underbrace{\sqrt{x+y}}_{b}) \cdot (\underbrace{\sqrt{a-b}}_{a} + \underbrace{\sqrt{x+y}}_{b})}{(\sqrt{a-b} + \sqrt{x+y})}$$

$$\cancel{= (\sqrt{a-b}}$$

$$= \frac{\overset{a^2}{(\sqrt{a-b})^2} - \overset{b^2}{(\sqrt{x+y})^2}}{(\sqrt{a-b} + \sqrt{x+y})} = \frac{(a-b) - (x+y)}{(\sqrt{a-b} + \sqrt{x+y})}$$

$$= \cancel{\frac{a-b-a-y}{\sqrt{a-b} + \sqrt{x+y}}} \cancel{= \frac{a-b-y}{\sqrt{a-b} + \sqrt{x+y}}}$$

$$= \frac{a-b-x-y}{\sqrt{a-b} + \sqrt{x+y}}$$

beispiel ($x = a$)

$$\frac{\cancel{a} - b - \cancel{a} - y}{\sqrt{a-b} + \sqrt{\cancel{x}+y}} = \frac{-b-y}{\sqrt{a-b} + \sqrt{x+y}}$$

Andere Regeln / Umschreibungsregeln

Vollständiges Video:

https://www.youtube.com/watch?v=NaA3-dOBGvA

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$n! \cdot n! = 2n!$

$n! \cdot (n+1) = (n+1)!$ Wichtig

$(n+1)! = n! \cdot (n+1)$

$n! = n \cdot (n-1)!$ Wichtig

$n \cdot a + a = (n+1)a$

$n+1^{n+1} = n+1^{n} \cdot n+1$ Wichtig

$5^{n+1} = 5^{n} \cdot 5$ Wichtig

$(2n-n)!\, n! = n! \cdot n!$

$(+) \cdot (+) = +$

$(-) \cdot (-) = +$

$(-) \cdot (+) = -$

$(+) \cdot (-) = -$

$\frac{(+)}{(+)} = +$

$\frac{(-)}{(+)} = -$

$\frac{(-)}{(-)} = +$

$\frac{(+)}{(-)} = -$

$\tan = \frac{\sin}{\cos}$ wichtig

$\cot = \frac{\cos}{\sin}$ wichtig

$\cos(0) = \cancel{0}\,1$

$\sin(0) = 0$

Umkehrfunktionen:

$\left.\begin{array}{l} \arctan = \tan \\ \arcsin = \sin \\ \arccos = \cos \\ \mathrm{arccotan} = \mathrm{cotan} \end{array}\right\}$ nicht so wichtig

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wichtige Grenzwerte:

$\lim_{x\to 0} \frac{\sin x}{x} = 1$ | $\lim_{x\to 0} \frac{x}{\sin x} = 1$ | $\lim_{x\to\infty} \frac{\ln x}{x^{\alpha}} = 0$

$\lim_{x\to 0^+} x^{\alpha} \cdot \ln(x) = 0$ | $\lim_{x\to 0} \frac{e^{x}-1}{x} = 1$ | $\lim_{n\to\infty} \left(1 + \frac{\alpha}{n}\right)^{n} = e^{\alpha}$

∞	∞	~~Pa~~ Polstelle
∞	$-\infty$	Polstelle mit VW.
1	1	Punktlücke
0	5	Sprungstelle (5)
0	$-\infty$	$\}$ einseitiges Polverhalten.
1	∞	